Scaling of critical connectivity of mobile *ad hoc* networks

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(Received 13 June 2008; published 16 December 2008)

In this paper, critical global connectivity of mobile *ad hoc* networks (MANETs) is investigated. We model the two-dimensional plane on which nodes move randomly with a triangular lattice. Demanding the best communication of the network, we account the global connectivity η as a function of occupancy σ of sites in the lattice by mobile nodes. Critical phenomena of the connectivity for different transmission ranges r are revealed by numerical simulations, and these results fit well to the analysis based on the assumption of homogeneous mixing. Scaling behavior of the connectivity is found as $\eta \sim f(R^{\beta}\sigma)$, where $R=(r-r_0)/r_0$, r_0 is the length unit of the triangular lattice, and β is the scaling index in the universal function $f(x)$. The model serves as a sort of geometric distance-dependent site percolation on dynamic complex networks. Moreover, near each critical $\sigma_c(r)$ corresponding to certain transmission range r, there exists a cutoff degree k_c below which the clustering coefficient of such self-organized networks keeps a constant while the averaged nearest-neighbor degree exhibits a unique linear variation with the degree k, which may be useful to the designation of real MANETs.

DOI: 10.1103/PhysRevE.78.066107 PACS number(s): 89.75.Hc, 89.20.Hh

I. INTRODUCTION

The mobile *ad hoc* network (MANET) [1,2] is a new sort of communication circumstance. It consists of many mobile nodes carrying out collective duty while nodes communicate with each other via wireless links. Neither central control authority nor intermediate services such as base stations for cellular mobile phones exist in the network. Each of its nodes needs to relay message to other participants within its limited transmission range so that a packet can arrive at its destination via successive internode transmissions (multihop links). The MANET changes its topology with time without prior notice since its nodes are free to move randomly [3] in doing traveling jobs. Therefore, to realize effective communication, it should self-organize into a dynamically stationary network by certain local protocols. The study of mobile *ad hoc* networks has attracted much attention recently due to their potential application in battlefields, providing disaster relief, outdoor assemblies, and other settings with temporal, inexpensive usage. Meanwhile it would represent a novel physical problem referring to critical phenomenon. Tens of protocols [4] have been proposed for the connectivity and other functions by designers. However, investigation on the property of the connectivity as viewed from statistical physics is still inadequate, which motivates the work in the present paper.

The theory of complex networks [5,6] can provide powerful tools to investigate the mobile ad hoc networks. Xie *et al.* [7] analyzed the formation of complex networks involving both geometric distance and topological degree of nodes. Sarshar *et al.* [8,9] noticed that while new nodes are added into the existing network, other nodes might leave the network rapidly and randomly. They presented results about the possible emergence of scale-free structure in *ad hoc* networks. However, they considered only static cases instead of analyzing the influence from the motion of nodes. Németh and Vattay [10] studied the giant cluster of such networks, and pointed out that the giant component size in the percolation could be described by a single parameter—the average number of neighbors of nodes. On the other hand, models of percolation on networks [10–13] were often employed to analyze spreading processes, especially epidemics with occupancy threshold p_c showing drastic transitions. New results from those models give out complementary conditions to statistical physics. In the present work on MANETs, we will find that the global connectivity of it can be treated as a new type of percolation on dynamical complex networks since nodes inside other ones' transmission circles could be directly linked while those outside them should be linked indirectly by multihop which is quite different from cases in traditional models of percolation.

A communication network may deliver meaningful services only if the network is well connected, or at least has a vast subset that is connected. Therefore, one goal of studying the *ad hoc* network is to find out how the network can maintain its connectivity [14]. In contrast to a previous study [10], we demand global connection of all nodes in the MANET for the best communication, which means an adequate condition to form a dynamically integrated network and a stricter case than usual site percolation [10–13,15,16]. For simplicity, we model the two-dimensional(2D) plane on which nodes move randomly as a triangular lattice with N vertices, so that we mimic round transmission ranges with discrete hexagons. We define the probability of global connection η as the ensemble average of n/n_0, where n is the number of moving nodes globally connected to the integrated network, and n_0 is the total number of them. Critical behavior of order parameter η is found to rely on both transmission range r and the occupancy σ (defined as n_0/N) of sites (i.e., vertices in the triangular lattice). Scaling behavior of the order parameter is verified in the form of $\eta \sim f(R^{\beta}\sigma)$, where R is the reduced

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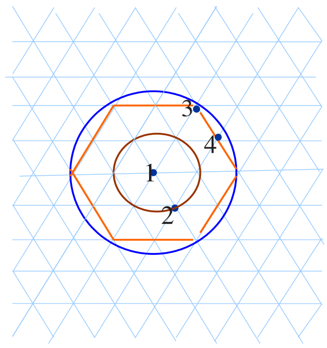

FIG. 1. (Color online) Triangular lattice with transmission range $r=r_0$ and $r=2r_0$, respectively, where r_0 is the length of every edge.

transmission range and β is the scaling index in the universal function $f(x)$. Moreover, at critical thresholds of occupancy, individual nodes self-organize into complex networks which display particular degree distribution [17], clustering coefficient [18], and averaged nearest-neighbor degrees [19] of such dynamic communication networks.

II. THE MODEL

To describe the case of self-organized communication of individual nodes, it is assumed that they move on a two-dimensional plane in a discrete way, and a simple dynamic *ad hoc* network model is proposed as follows. On the two-dimensional triangular lattice (see Fig. 1) of size L (i.e., the number of the sites in a row of it), individual nodes are assumed to distribute randomly on the sites of it. In our work, the total number of the sites are chosen as $N=L^2$ with $L=200$. And, under periodic boundary condition, we restrain the motion of nodes along edges between neighboring sites. At the initial time step, assign n_0 nodes to the sites in the triangular lattice randomly. Every site of such a lattice can be occupied by only one node or nothing. Considering the dynamic topology of the ad hoc network, we suppose that, at every time step, each node can move randomly in one of the six directions to its neighbor site if it is not occupied. Two nodes in the *ad hoc* network can communicate with each other if the distance between them is less than the minimum of their transmission ranges r [20,21]. To simplify our study, we assume that all the mobile nodes have the same fixed transmission power [22,23]. Therefore they all have equal transmission range r for valid communication of the whole network since the area of the effective transmission circle (hence r^2) is proportional to it. Note that neither self-connection nor multiple edge is allowed in the network: (1) A node should not communicate with itself and (2) technically there is no sense to open another communication channel between any two nodes if they are already direct communication neighbors. The motion of nodes at a certain value of occupancy forms different configurations of site occupation on the lattice, which serves as the ensemble for the calculation on the global connectivity (i.e., the probability of the global connection) and other averaged quantities.

The transmission range r is an important parameter in the designation of *ad hoc* networks, since it is vital to keep the network globally connected. Proper adoption of range r could minimize energy consumption and reduce mutual interference between nodes. Under certain node occupation density on the lattice, when nodes have large transmission range r so that their service area overlap enough to ensure the global connection, packets transmission of nodes will be depressed fatally because of the mechanism of medium access control (MAC) which means that nodes inside the circle of a transmitting node cannot emit a message simultaneously. Moreover, this scenario consumes much energy which is vital to the living time of moving nodes. On the contrary, transmission area could hardly overlap each other when nodes have too small transmission range r. Such a power-control puzzle is always a dilemma for operating a MANET. We attempt to treat it through the approach of statistical physics in addition to many technical protocols [14]. What we considered here are, under which conditions almost all nodes would be surely connected globally? And what is the critical behavior of global connectivity?

III. SCALING PROPERTY OF GLOBAL CONNECTIVITY

An intuitive way to ensure the best communication by multihop linking is to increase the occupancy of sites on the lattice by mobile nodes, while this changes the energy consumption of the whole network. In the framework of traditional percolation problems, continuously increasing site occupancy will pass threshold of site percolation. To our knowledge, the term "site" can be used dually: one means a block [24], the other means a vertex [25]. Obviously our usage belongs to the latter one. Two sites are geometric neighbors when two occupied vertices are directly linked by a common bond (edge). Therefore, an indirect connection between any two sites means that a path exists from one to the other neighbor by neighbor. In the present model, however, every node is located at the center of its own transmission range $r=zr_0$ (z is a positive integer and r_0 is the length of an edge of any minimal triangle). All the other nodes at sites inside this circle connect with it directly. Taking the circle $r=2r_0$ (blue in Fig. 1) as an example, nodes 2, 3, and 4 inside the inscribed hexagon (orange in Fig. 1) of it are direct neighbors of node 1. Therefore, neighbors here are determined by the transmission range and in the sense of topological connection, just what occurred in complex network models [11,12], which distinguishes them from those in traditional 2D site percolation. It is a special case of topological correlation valid within certain geographic distance [26–30]. The triangular lattice in our model is a beneficial setting to describe moving nodes and for discrete calculation of network parameters. Moreover, in a percolation problem one always focuses on the probability for a site to be included in the giant component which just extends from a border to its opposite one in a lattice with finite size. However, for an *ad hoc* network bearing search, rescue, tracing, or precise attack, the performance may rely on the message and behavior of a few nodes, even a single moving node. It may demand global connection of all nodes, which is quite different from the percolation problem which leaves many nodes scattering outside the giant component. For instance, when nodes in a MANET are assigned moving duty of multiple-target attack, they need to share messages with each other to recognize everyone's target, respectively, to estimate attack results thus to avoid waste of ammunition. Inversely, when attack comes from outside, it should be noticed to every member immediately by MANET for collective defence.

The order parameter η, i.e., the global connectivity of the MANET, is calculated with burning algorithm [31]. The evolution of it should rely on the occupancy σ of the sites, and η enhances for fixed r when σ increases. The ratio of the enhancement depends on the number n of the nodes which have been connected into the largest dynamic network at that time step, and it also depends on the number of the disconnected nodes, i.e., $(n-n_0)$, based on the assumption of homogeneous mixing [32] of randomly moving nodes. Therefore, we have

$$\frac{d\eta}{d\sigma} \propto n(n_0 - n). \tag{1}$$

Using the definition of η and getting the effect of the transmission range included, we arrive at

$$\frac{d\eta}{d\sigma} = g(r)\,\eta(1-\eta), \tag{2}$$

where $g(r)$ is the function of transmission range r. This equation can be solved with the uniform initial condition $\eta(\sigma \to 0) = \eta_0$:

$$\eta(\sigma) = \frac{\eta_0}{\eta_0 + (1-\eta_0)e^{-g(r)\sigma}}. \tag{3}$$

In Fig. 2, simulation results for the global connectivity as a function of the occupancy of the sites on the triangular lattice are illustrated. They are in good agreement with the analytical result of Eq. (3) under the condition of dimensionless function $g(r) \sim r/r_0$. Actually, it is naturally expected by dimension analysis on the exponent in the denominator: $g(r)$ should have no dimension since occupancy σ is dimensionless. The difference between simulation and analytical results at bottom parts can be attributed to the deviation from homogenous assumption by the distribution of nodes at discrete sites on the triangular lattice and size effect.

Numerical results display the critical behavior of global connectivity, i.e., they show its drastic transitions occurring at critical values σ_c for different transmission ranges. When occupancy σ passes σ_c, the dynamically moving nodes self-organize from a disconnected state to a surely globally connected one. For our triangular lattice model, σ_c=0.37, 0.21, 0.13, 0.09, and 0.065 for $r=2r_0$, $3r_0$, $4r_0$, $5r_0$, and $6r_0$, respectively. Obviously, various critical values of node occupancy σ_c are required to ensure global connection for different transmission ranges in MANETs, which means that we can also inversely choose proper transmission range to minimize energy consumption of the network for different density of nodes on the lattice.

It is natural to rescale $\eta(\sigma, r)$ into a universal scaling function from direct observation of Fig. 2(a). We have

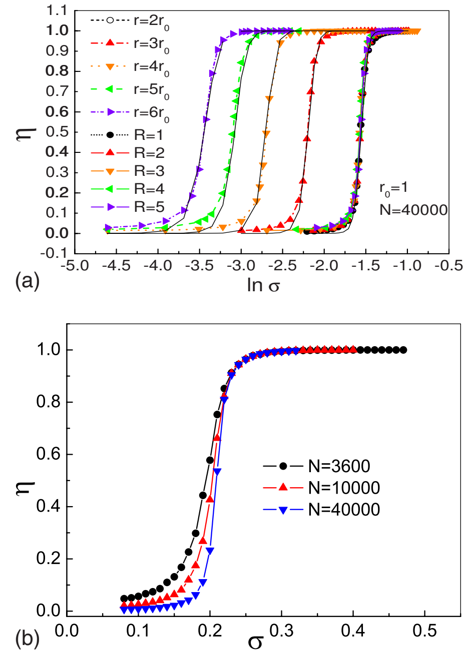

FIG. 2. (Color online) (a) Comparison between theoretical analysis and numerical simulation on the probability of global connection as a function of σ for various transmission ranges r. Thin full lines show analytic results given by formula (3). Color symbols represent transition behavior of critical global connectivity of mobile nodes for different transmission ranges. $\eta(\ln \sigma)$ curves collapse into the same one with $r=2r_0$ under the scaling of formula (4). (b) Critical connectivity under $r=2r_0$ with different sizes of the lattice. Hereafter all simulation results are averaged over 300 realizations of configurations formed by moving nodes on the triangular lattice.

$$\eta \sim f(R^{\beta}\sigma) \tag{4}$$

with reduced transmission range $R=(r-r_0)/r_0$ and the scaling index β of the universal function $f(x)$, respectively. Transition curves are also drawn in Fig. 2(a) to show rescaling process. One can see that calculated curves for all r collapse into the one with $r=2r_0$, and the index $\beta=-0.49$ gives perfect convergence of all the curves. This provides the evidence that the transitions at $\sigma_c(r)$ are really critical phenomena. As a comparison, percolation exponent of 2D triangular lattice is -0.50 [33], which means exponent of the present MANET model only deviates slightly from standard case although the mechanism is quite different from it. And we have also checked a similar setting of 2D plane described by a square lattice, the scaling function keeps the same form but with different exponent (i.e., -0.61) [34], which means the designed settings of 2D lattices bias the features of MANET models. We prefer triangular lattice to square one since it can form hexagonally symmetric cells which is more approaching circular transmission ranges of nodes. Figure 2(b) shows η versus σ for different sizes of MANETs with $r=2r_0$. The critical value σ_c is independent of the sizes of lattices, which is also valid for different transmission ranges. Near critical points $\sigma_c(r)$, nodes with autonomic communication self-organize into time-varying complex networks which are reminiscent of directed dynamic small-world network

(DDSWN) model [35] but with different scaling variables. Indeed, the ratio of the number of nodes receiving message to the total number of nodes should vary in the same way as that model provided the nodes move at the same speed, have uniform transmission range and relay message without delay, which will be discussed under another title. But in the present work we check global connectivity with a burning algorithm, assuming that "combustion" (similar to message spreading out) is much faster than variation of topological structure. Moreover, the feature of transmission range dependence distinguishes itself from DDSWN model. It is also noticeable that Hu and Chen [36] investigated scaling functions for bond random percolation on honeycomb lattices with different aspect ratios. By comparison, the present model is pertaining to globally connected network checked with combustion algorithm on a triangular lattice although we always pay attention to the hexagonal cell within the circular communication range.

IV. TOPOLOGICAL PROPERTIES OF MANET

When the occupancy of nodes just exceeds the value of σ_c for certain transmission range r, they are found to self-organize into a dynamically stationary network with our simulations. We can characterize the connection of an ad hoc network with parameters of complex networks. The simplest and the most intensively studied parameter is degree distribution $p(k)$ [17] because it may govern fundamental properties of the system. Degree k of a node, as well known, is the total number of its topological edges connecting with others. The dispersion of node degree is characterized by the distribution function $p(k)$ which gives the probability that a randomly selected node has exactly k edges. In the present paper, we study parameters of the network when it consists of almost all the nodes of MANETs together. The degree distribution $p(k)$ follows a Poisson distribution which is different than that in Ref. [7–9]. Figure 3(a) shows the ensamble averaged degree distributions for all simulated transmission ranges r when the global connectivity of nodes is above 0.9995. A cutoff degree k_c=17 appears in it, which means that the probability for any node to have degree $k > k_c$ is very low. Figure 3(b) shows cutoff degree k_c versus varying occupancy σ for $r=4r_0$. The averaged degree of the whole network can be obtained from direct observation of Fig. 1, that is,

$$\langle k\rangle = 3r(r+1)\sigma, \tag{5}$$

where $r=zr_0$ as mentioned above. We recognize MANET from this kind of Poisson-like distribution as random networks or small-world networks. By the way, the global connectivity η is also a single-variable function of average degree $\langle k\rangle$ for certain r, which is qualitatively similar to the behavior of component size S in Ref. [10] since σ is proportional to $\langle k\rangle$ in such cases.

Clustering coefficient C [18] and k_{NN} [19], the averaged nearest-neighbor degree of nodes depicts complex networks as viewed from correlations. For a node i, clustering coefficient C_i can be defined as the fraction of pairs of node i's neighbors that are also neighbors of each other in the topological sense. $C(k)$ of the network is the clustering coefficient averaged over nodes with the same value of degree k. In our case, $C(k)$=0.52, 0.55, 0.56, 0.57, and 0.57 for transmission range $r=2r_0$, $3r_0$, $4r_0$, $5r_0$, and $6r_0$, respectively, and keep invariant for $k \leqslant k_c$. The k-independent behavior of $C(k)$ can be understood from the symmetry of hexagonal cells and homogeneous distribution. Let us scrutinize the hexagonal cell (orange in Fig. 1) for $r=2r_0$ as an example. Site 1 in it has 18 neighbors (assuming full occupation). Therefore, the largest possible number of closed topological triangles in the sense of complex network should be C_{18}^2 which serves the denominator of the clustering coefficient of it. Linking occupied sites 1 and 2, we search for the third one with the distance less than $2r_0$ to both of them in the hexagonal cell, which makes seven triangles. Linking occupied sites 3 and 4 to the center 1, makes 8 and 12 triangles, respectively. Therefore, all equivalent sites $[(7+8+12)\times 6/2]$ in the cell make 81 closed triplets under the constraint of maximum distance $2r_0$, and yields the clustering coefficient of the site as $81/C_{18}^2$=0.53. Under the assumption of homogeneous mixing, this is also valid for any occupancy or averaged $\langle k\rangle$ since we only need to multiply both the numerator and the denominator of it by local σ simultaneously, so that we have the same $C(k)$ for small $k(k \leqslant k_c)$. The horizontal line for $k \leqslant 17$ (see Fig. 4) of $C(k)$ indicates particular constant clustering coefficient of MANETs for degrees occurring in high possibilities. Here value k_c reflects the limit case of connection, and it may be pertinent to the structure of the hexagonal cell imbedded in the triangular lattice. The high value (above 0.5) of $C(k)$ implies that we have dynamic small world networks at critical points, and there is large redundancy in communication if only strategy of increasing occupancy of sites is adopted. The averaged nearest-neighbor degree of node i is simply $k_{NN,i}=\Sigma_j k_j/k_i$, where j is a neighbor of node

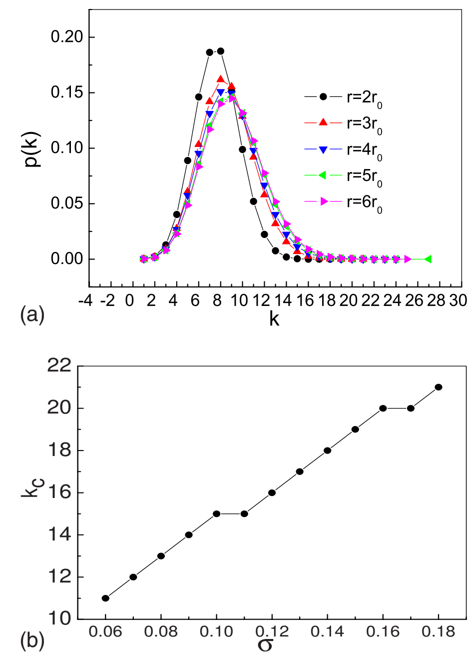

FIG. 3. (Color online) (a) The degree distribution $p(k)$ of MANETs with η=0.9995 for different transmission ranges r. (b) Cutoff degree k_c versus varying occupancy σ for $r=4r_0$.

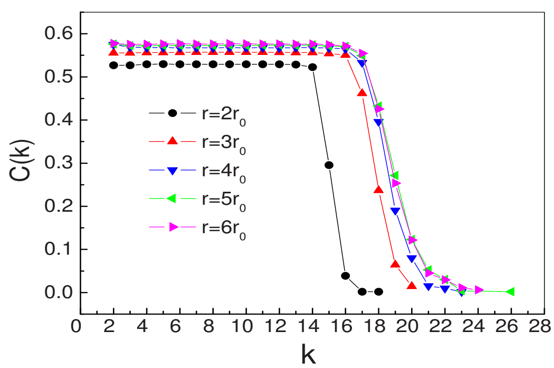

FIG. 4. (Color online) The clustering coefficient $C(k)$ for different transmission ranges r on the triangular lattice with $\eta=0.9995$.

i. And $k_{NN}(k)$ can be accounted as the function of degree k in the following form: $k_{NN}(k)=\Sigma_{k_i\in V}k_{NN,i}/\Sigma_{k_i\in V}1$, where V is the subset of nodes with the same degree k. From Fig. 5 we can see that the curve of k_{NN} versus k suggests an empirical formula in the linear form for our MANET, that is,

$$k_{NN}(k) = b(r) + C(k)k, \tag{6}$$

where $b(r)$ is a k-independent constant. The positive assortativity [i.e., increase behavior of $k_{NN}(k)$] gives the particular feature distinguishing it from most other technical networks [37–39]. The appearance of tails in large degrees ($k>17$) (see Figs. 4 and 5) is due to very low probabilistic occurrence in the simulation on 300 realizations.

V. DISCUSSION AND CONCLUSIONS

The present model describes the upper bound of occupation density σ_c, above which we are sure of global connection, adaptive to homogeneous transmission range r. However, global connection is possibly realized since motion of nodes could cause some extra effects of dynamical links or trims with certain protocols. With particular protocols σ_c is hopeful to decrease. Actually we demand instantaneous percolation for the configuration at every moment which enables us to compare the result with those of previous standard models. Corresponding to certain transmission range r we have critical density σ_c below which some particular protocols are needed to guarantee spontaneous global connection. Therefore, it is adequate but not necessary condition for the transition to well connected phase. A recent work by Gonzalez *et al.* [40] inspires a clue to deal with moving connection effect of nodes. Agents accumulate degree when they move in a continuous medium with a particular collision rule. It refers to social contacts and percolation occurs in a high dimensional Euclidean space which could be projected onto a 2D one. On the contrary, the present MANET model counts degree for each configuration at every moment. It refers to real physical distance on a 2D triangular lattice and should be accounted as a novel type of percolation. Anyway their model could be a reference for further work on MANET although it is not straightforward to analogue one another now. Moreover, investigations on cubic lattice even a complex network itself as the background would also be interesting to do. With both long-range and short-range dynamical links it would mix up the effects from topology with that of distance-dependent ones, therefore new behaviors could be expected.

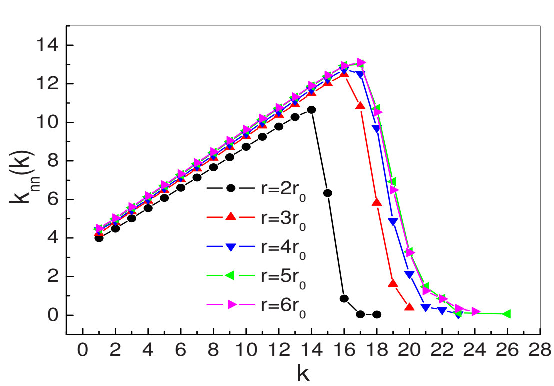

FIG. 5. (Color online) The averaged nearest-neighbor degree of nodes $k_{NN}(k)$ for different transmission ranges r on the triangular lattice with $\eta=0.9995$.

Interestingly, the model can be thought of as a special example for the process of coevolutionary competitive exclusion [41]: taking sites as the state variable of moving nodes, each node tends to compete for links to others, which is demanded by the global connection. Only nodes within other ones' transmission ranges and with the degrees of both ones less than k_c could successfully gain links, which constitutes threshold conditions of the coevolutionary network.

In summary, we present a model for mobile *ad hoc* networks by considering uniform transmission range of nodes and assigning moving nodes randomly on the plane of the triangular lattice. With both analysis and simulations, critical behaviors of global connectivity versus node occupancy of sites are found for various transmission ranges. The order parameter scales with transmission ranges, but behaves differently than other models. This forms a new sort of percolation in dynamic complex networks pertaining to geometric distance. Moreover, cutoff degree k_c, invariant clustering coefficient and linear assortativity as functions of degree are found for self-organized dynamical complex networks near critical global connectivity. Our model suggests that transmission range of nodes and average occupancy on the plane should adapt to each other to balance minimization of energy consumption with the global connectivity. In fact, nodes relaying message consume energy continuously. Transmission range of each node usually reduces with time relating to its job load. Therefore, the present model is applicable only for the routing strategy of broadcasting [42]. A more practical model should include randomly distributed nodes with changing transmission ranges. Meanwhile, nodes are not necessary to move along edges of the triangular lattice. Therefore, a random graph model without any lattice is necessary for better investigation on mobile ad hoc networks, which leaves our further work for the future.

ACKNOWLEDGMENTS

We feel indebted to Shi-Jie Xiong, Da-Ren He, and Bing-

Hong Wang for very valuable discussion and direct revision to the manuscript. We also acknowledge partial support from the National Natural Science Foundation of China (NNSFC) under Grants No. 70471084, No. 10775071, No. 10635040, and No. 60676056. C.P.Z. and B.H.W. thank the hospitable accommodations of Bao-Wen Li in NUS. C.P.Z. thanks Xiang-Tao Fan, Y.-C. Lai, and B. J. Kim for checking the English of the manuscript. We also acknowledge the support by the National Basic Science Program of China Projects No. 2006CB921803 and No. 2006CB705500.

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